CSET102

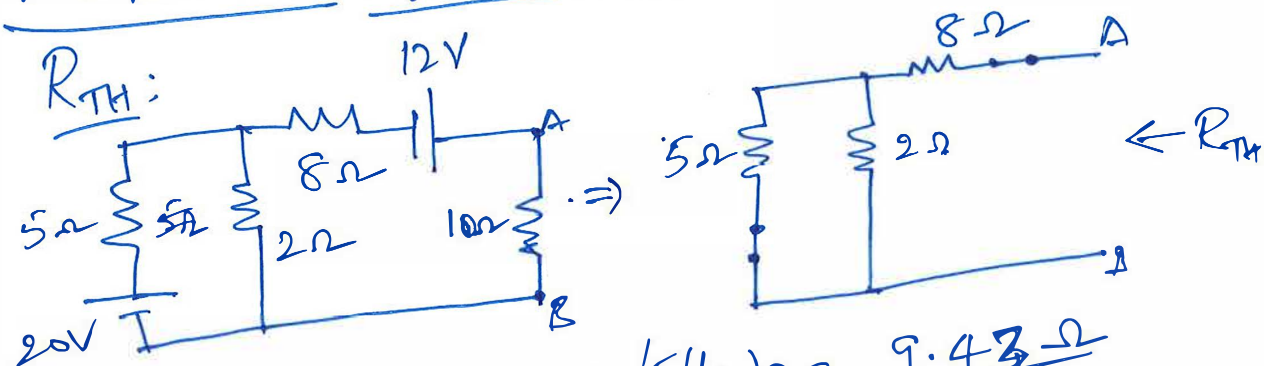
TUTORIAL SHEET-06
SOLUTIONS

1 & 2

Fig. 1

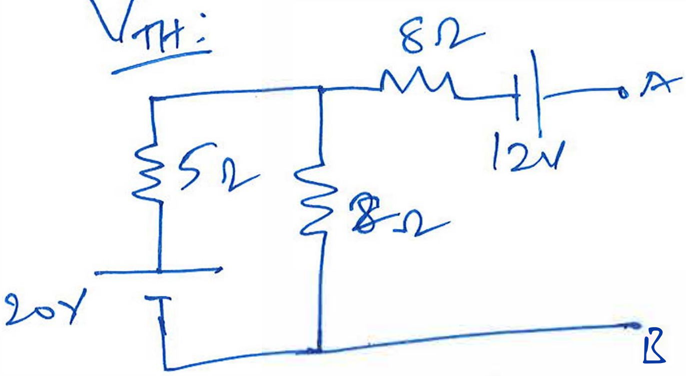
Thevenin's Equivalent:

R_{TH}:

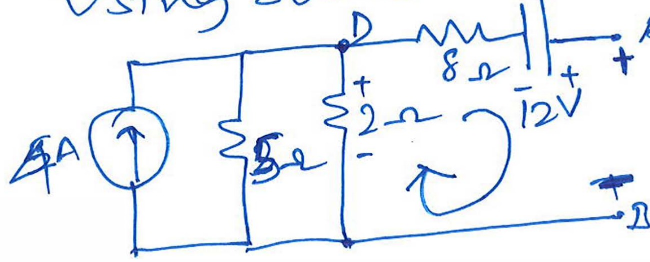


$$R_{TH} = 8\,\Omega + (5 \| 2)\,\Omega = 9.43\,\Omega$$

V_{TH}:



Using source transformation



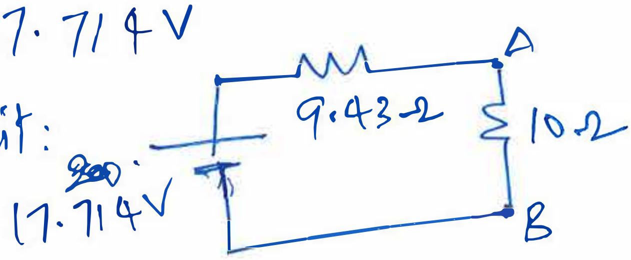
$$I_{2\Omega} = \frac{4 \times 5}{2+5} = 2.86\,A$$

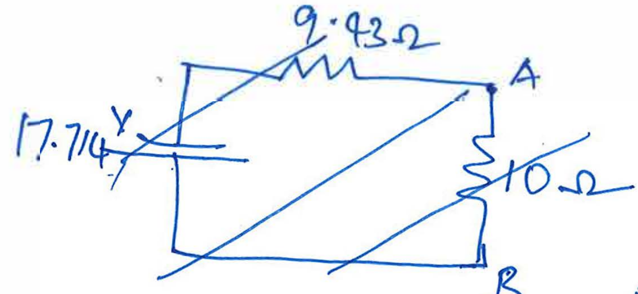
Apply KVL in loop ABDA $12 - V_{AB} + 2 \times 2.86 = 0$

$$V_{AB} = +17.714\,V = V_{TH}$$

$$V_{TH} = V_{AB} = +17.714\,V$$

Equivalent circuit:





$$I_L = \frac{17.714}{9.43+10} = 911.7\,mA$$

$$V_L = \frac{17.714 \times 10}{19.43} = 9.117V \qquad P_L = V_L \cdot I_L = \underline{8.31\,W}$$

As $R_L \neq R_{TH}$, maximum power is transferred.

Maximum power is transferred when $R_L = R_{TH}$

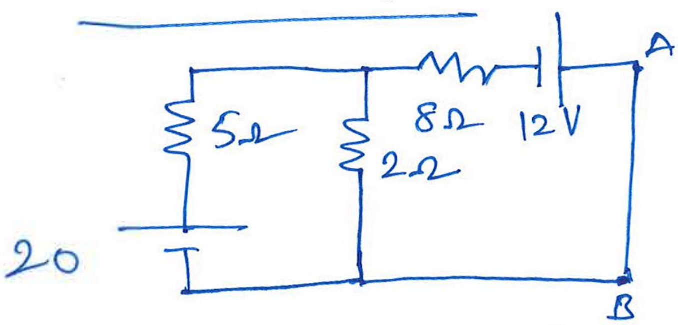
$$P_{Max} = V_L\big|_{R_L=R_{TH}} \cdot I_L\big|_{R_L=R_{TH}} = \frac{V_{TH}}{2} \cdot \frac{V_{TH}}{2R_{TH}} = \frac{V_{TH}^2}{4R_{TH}}$$

$\underline{P_{Max} = 8.32\,W}$

Norton Equivalent:

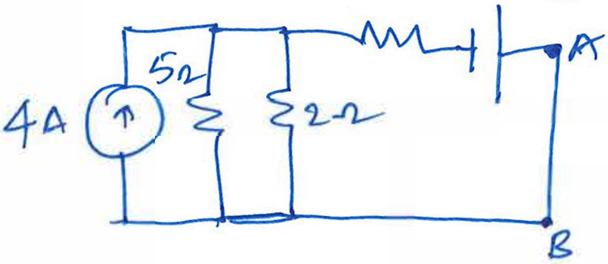
$R_N = R_{TH} = 9.43\Omega$

to find I_N:



using source transformation,

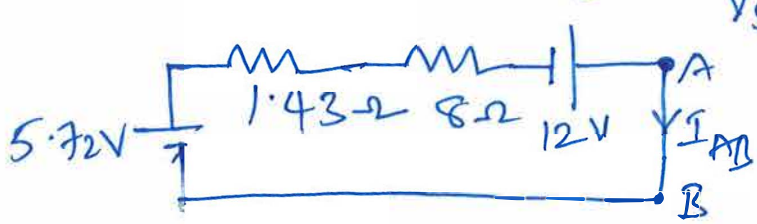
$$I_S = \frac{20}{5} = 4A$$



$5 \| 2 = 1.43\Omega$

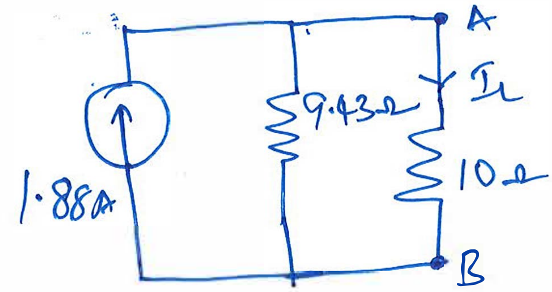
Using Source transformation

$V_S = 4 \times 1.43 = 5.72V$



$$I_{AB} = \frac{12+5.72}{1.43+8} = 1.88A$$

$$I_N = I_{AB} = 1.88\,A$$



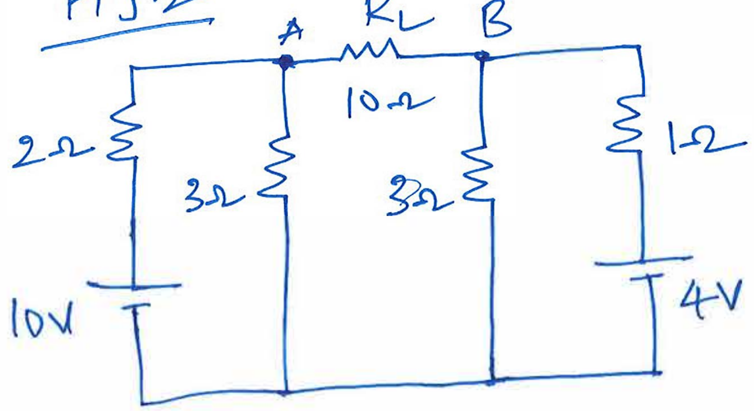
$$I_L = \frac{1.88 \times 9.43}{10 + 9.43} \sim 912\,mA$$

$$V_L = 9.12\,V$$

$$P_L = I_L \cdot V_L = \underline{8.32\,W}$$

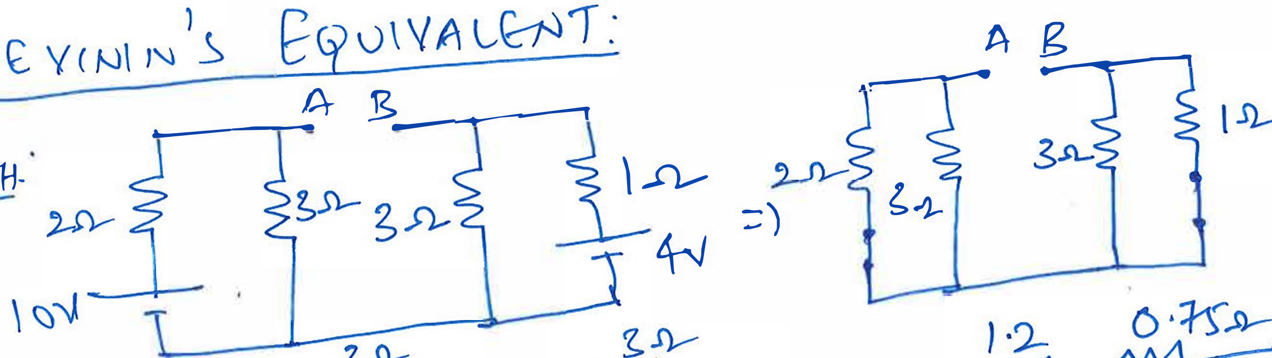
(small errors are due to rounding off)

Fig. 2

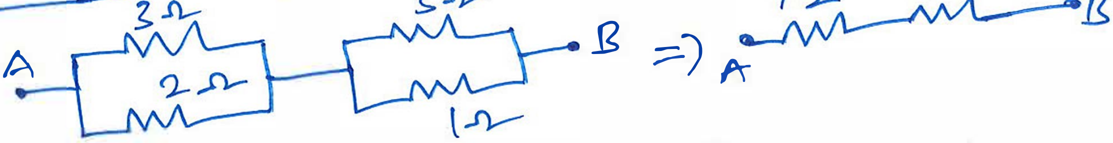


THEVENIN'S EQUIVALENT:

R_{TH}:

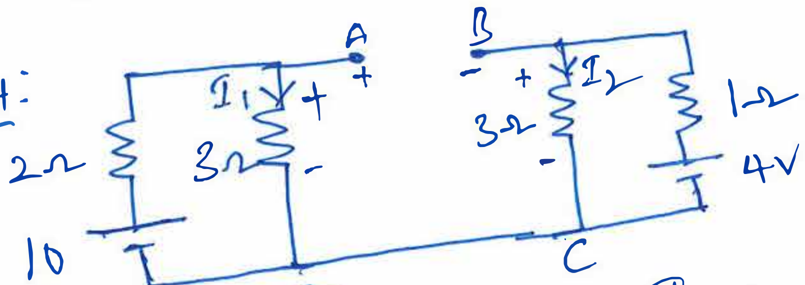


R_{AB} =



$R_{AB} = 1.95\,\Omega$ $R_{TH} = R_{AB} = 1.95\,\Omega$

V_{TH}:



$$I_1 = \frac{10}{5} = 2A \qquad I_2 = \frac{4}{4} = 1A$$

Applying KVL in the loop ABCA,

$$V_{AB} = 3\times 2 - 3\times 1 = 3V$$

$$V_{TH} = V_{AB} = \underline{3V}$$

$$V_L = \frac{3\times 10}{10+1.95} = \underline{2.51V}$$

$$I_L = \frac{V_L}{R_L} = 0.251A$$

$$P_L = V_L \cdot I_L = 0.63W$$

Power deliverd to load is maximum when $R_L = R_{TH}$

for $R_L = R_{TH}$, $V_L = \frac{V_{TH}}{R_L+R_{TH}} = \frac{V_{TH}}{2} = 1.5V$

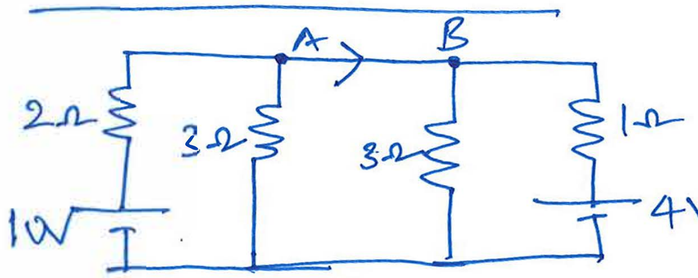
I_L for $R_L = R_{TH}$: $I_L = \frac{V_{TH}}{R_L} = \frac{1.5}{1.95} = 0.77A$

$$\underline{P_{L_{MAX}} = 1.154W}$$

NORTON's EQUIVALENT :

$$R_N = R_{TH} = \underline{1.95\Omega}$$

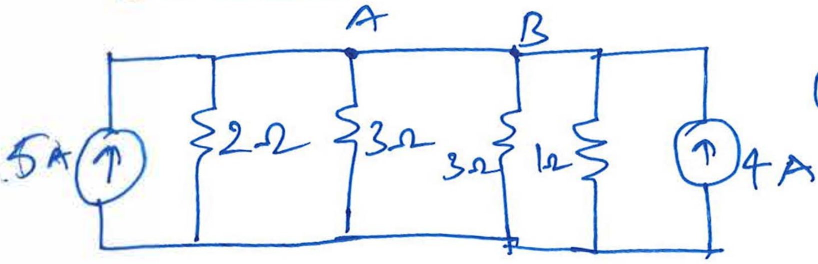
To calculate I_N:



Using Source transformation

$$I_{S_1} = \frac{10}{2} = 5A$$

$$I_{S_2} = \frac{4}{1} = 4A$$

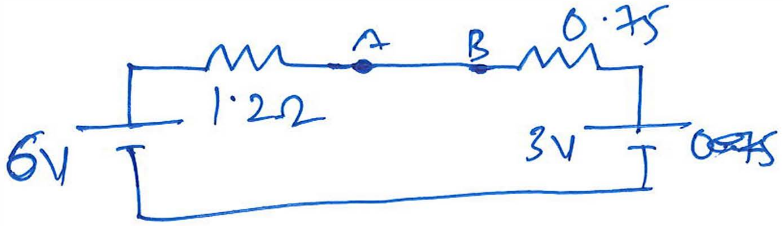


$2 \| 3 = 1.2\Omega$

$3 \| 1 = 0.75\Omega$

Using source transformation,

$V_{s1} = 5A \times 1.2 = 6V$ $V_{s2} = 4A \times 0.75 = 3V$



$I_{AB} = \frac{6-3}{1.95} = 1.54A$

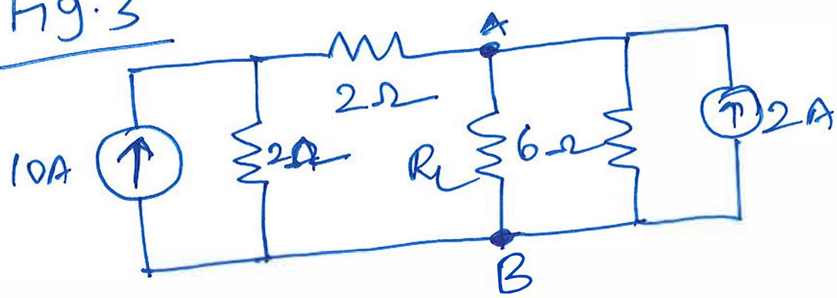
$I_N = I_{AB} = 1.54\ A$

$I_L = \frac{1.54 \times 1.95}{11.95} = 0.251A$

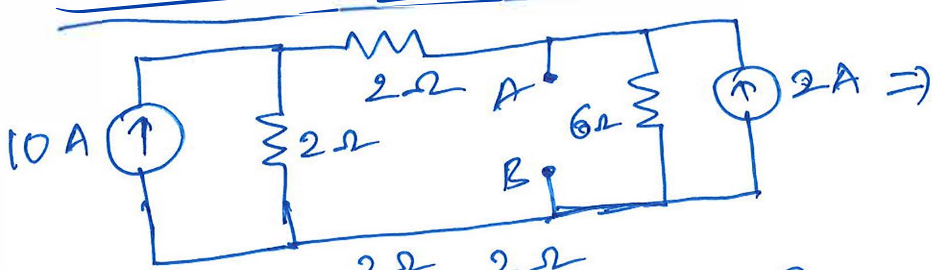
$V_L = 2.51V$

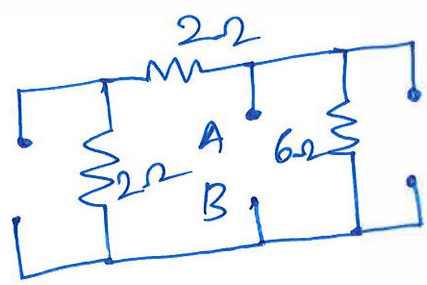
$P_L = V_L \cdot I_L = \underline{\underline{0.63W}}$

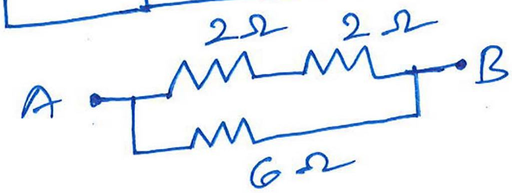
Fig. 3



THEVININ'S EQUIVALENT







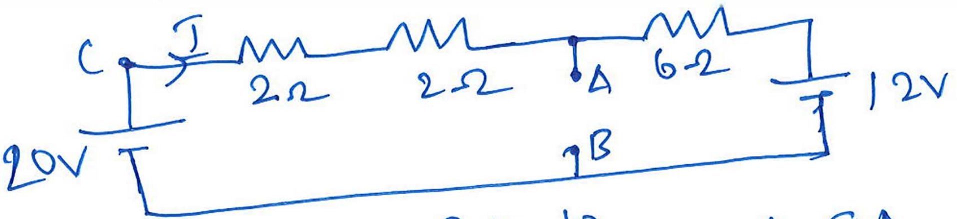
$R_{AB} = 4 \| 6 = 2.4\Omega$

$R_{TH} = R_{AB} = 2.4\Omega$

To find V_{TH}:

Using Source transformation,

$V_{S_1} = 10 \times 2 = 20V$ $\quad V_{S_2} = 2 \times 6 = 12V$

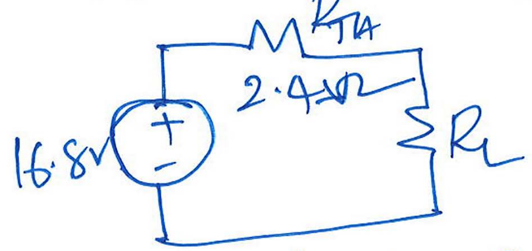


$$I = \frac{20-12}{2+2+6} = 0.8A$$

Applying KVL, for the loop ABCA,

$$20 - I(2+2) = V_{AB} = 20 - 0.8 \times 4 = 16.8V$$

$V_{AB} = 16.8V$ $\quad \underline{V_{TH} = V_{AB} = 16.8V}$



for maximum power transfer $R_L = R_{TH} = 2.4\Omega$

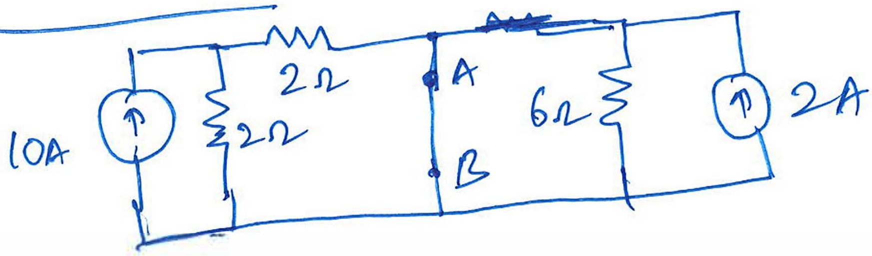
$$V_L = \frac{16.8 \times 2.4}{2.4 + 2.4} = 8.4V$$

$$I_L = \frac{8.4}{2.4} = 3.5A \qquad P_L = \cancel{28.04}\ 29.4W$$

Norton's Equivalent:

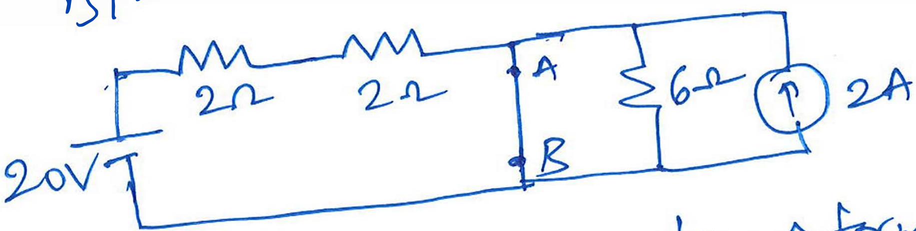
$R_N = R_{TH} = 2.4\Omega$

To find I_N:



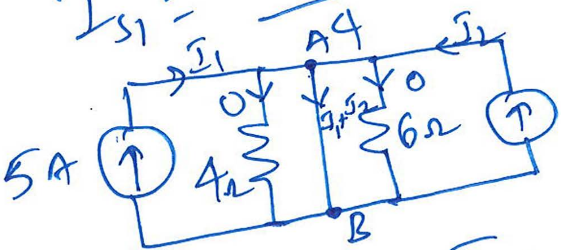
Applying source transformation,

$V_{S1} = 10 \times 2 = 20V$



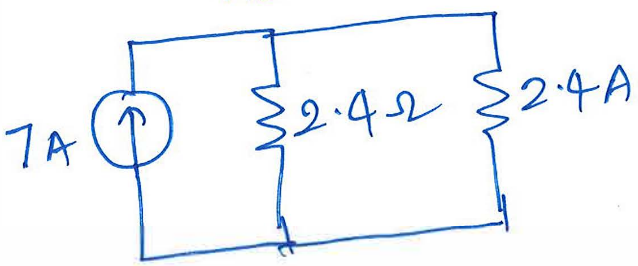
Applying source transformation,

$I_{S1} = \frac{20}{4} = 5A$



$I_1 + I_2 = 7A$

$I_{AB} = 7A = I_N$

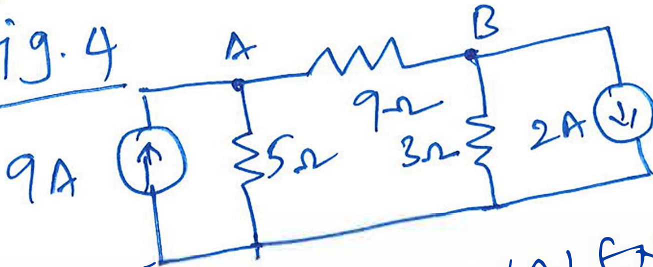


$I_L = \frac{7 \times 2.4}{4.8} = 3.5A$

$V_L = I_L R_L = 8.4V$

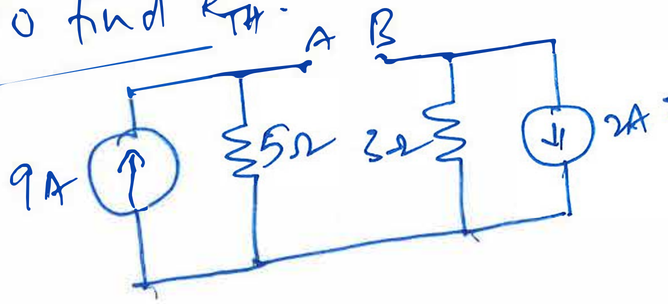
$P_L = \underline{29.4\ W}$

Fig. 4

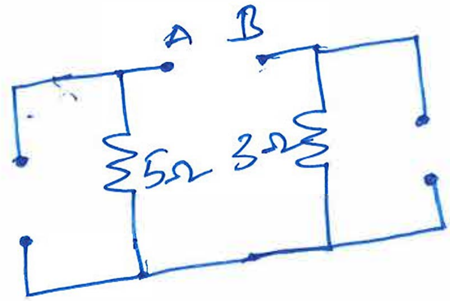


THEVININ's EQUIVALENT CIRCUIT :

To find R_{TH} :

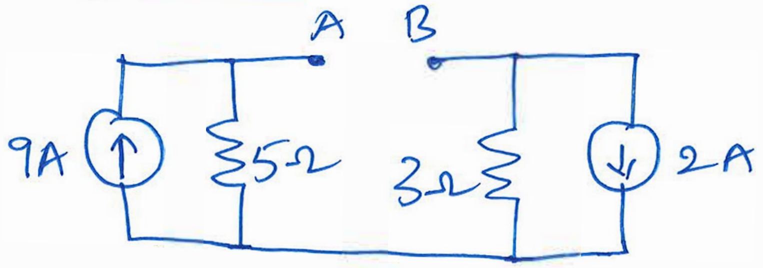


$\Rightarrow$



$R_{AB} = 8\Omega$ $\quad R_{TH} = R_{AB} = \underline{8\Omega}$

To find V_{TH}:



Using Source transformation

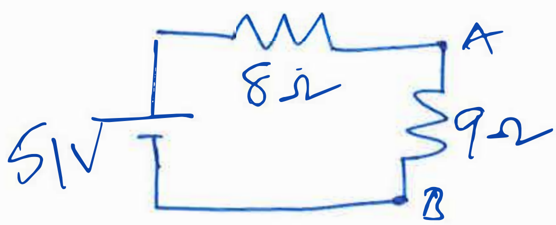
$V_{S_1} = 9 \times 5 = 45V$

$V_{S_2} = 2 \times 3 = 6V$

Applying KVL for the loop ABCA,

$45 - V_{AB} + 6 = 0 \quad V_{AB} = \underline{51V}$

$V_{TH} = V_{AB} = \underline{51V}$



$V_L = \dfrac{51 \times 9}{9+8} = 27V$

$I_L = 27/9 = 3A$

$P_L = V_L I_L = 81W$

P_L is maximum when $R_L = R_{TH}$

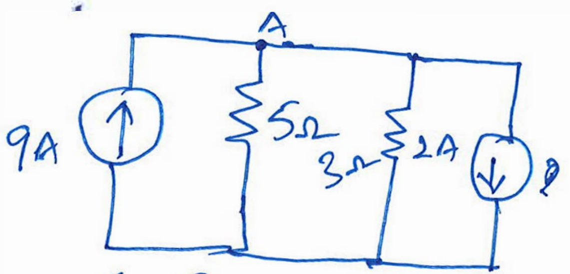
Then $P_{L,max} = \left.\dfrac{V_L \cdot R_L}{R_L + R_{TH}}\right|_{R_L = R_{TH}} \cdot \left.\dfrac{V_L}{R_L + R_{TH}}\right|_{R_L = R_{TH}}$

$= 25.5 \times \dfrac{25.5}{8} = \underline{81.28W}$

NORTON'S EQUIVALENT

$R_N = R_{TH} = 8\Omega$

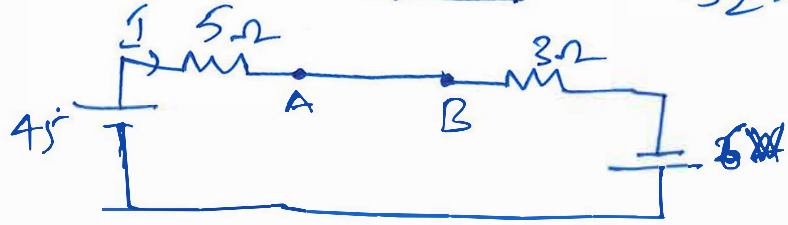
To find I_N:



Using source transformation

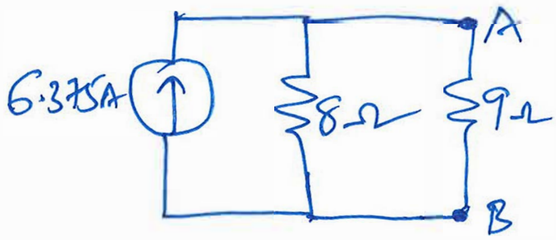
$V_{s_1} = 9\times 5 = 45V$

$V_{s_2} = 2\times 3 = 6V$



$I = \dfrac{45+6}{8} = 6.37$

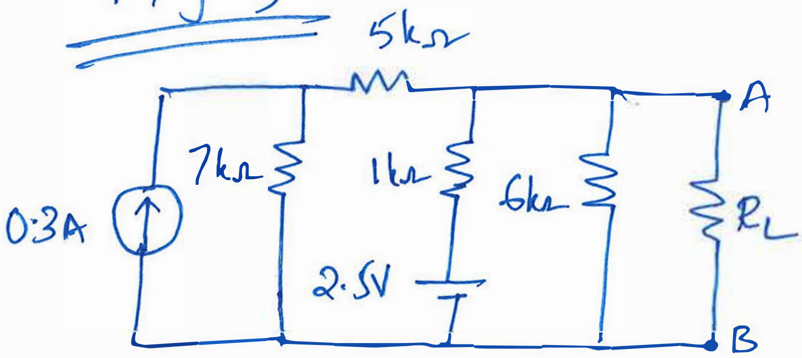
$I_N = 6.37$



$I_L = \dfrac{6.375\times 8}{8+9} = \underline{3A}$

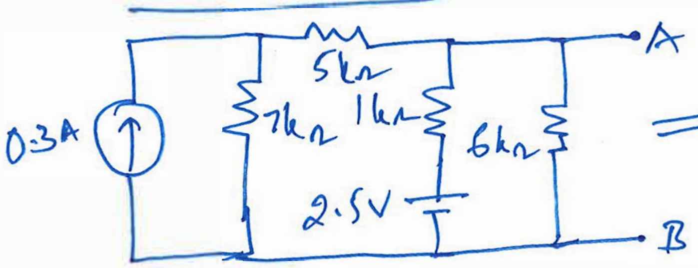
$V_L = I_L\cdot R_L = 3\times 9 = \underline{27V}$

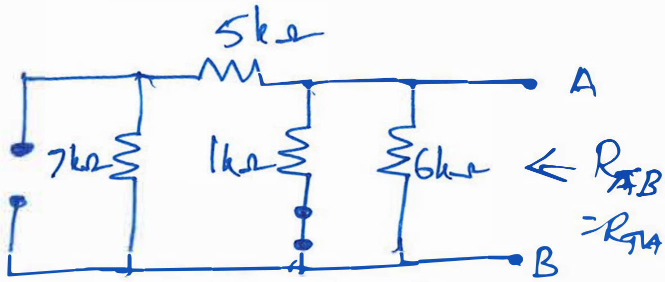
Fig. 5

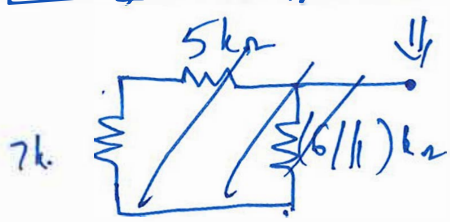


THEVININ'S EQUIVALENT

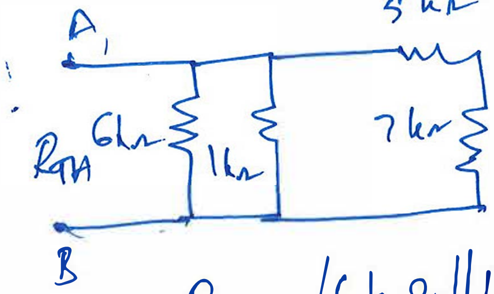
To find R_{TH}



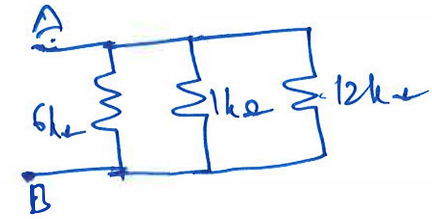




$6k||1k = \cancel{857\Omega}$



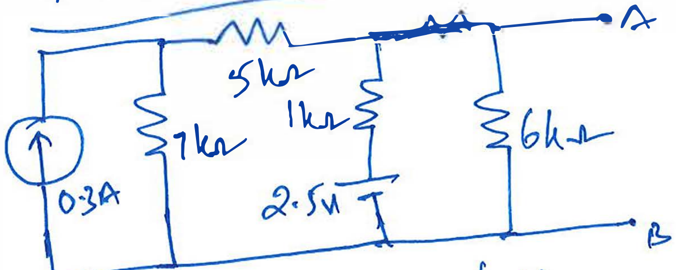
⇒



$$R_{AB} = (6k\Omega \,||\, 1k\Omega \,||\, 12k\Omega) = 800\Omega$$

$$R_{TH} = R_{AB} = 800\Omega$$

To find V_{TH}



$V_{TH} = V_{AB} = V_{6k\Omega}$ = Voltage across 2.5V source and 1kΩ.

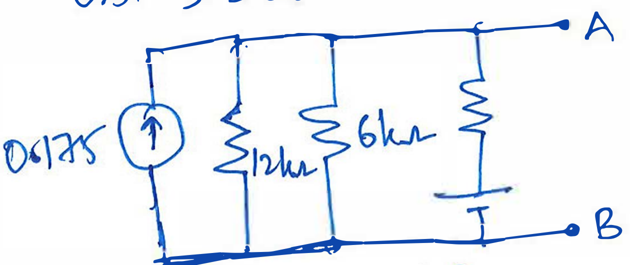
Using Source Transformation

$V_{S_1} = 0.3 \times 7k = 2100\,V$

⇓

$$I_{S_1} = \frac{2100V}{12k\Omega} = 0.175A$$

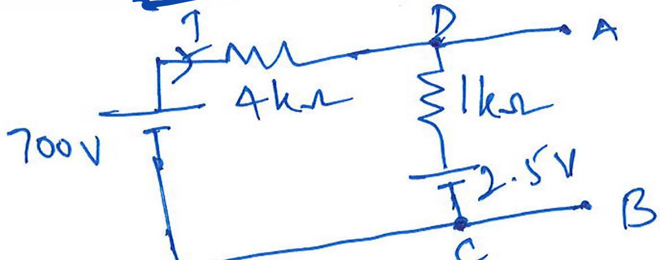
Using Source transform;



$12k\Omega \,||\, 6k\Omega = 4k\Omega$

Using Source transformation,

$V_{S_1} = 0.175A \times 4k\Omega = 700V$

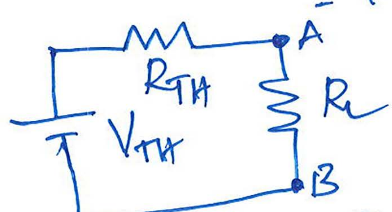


$$I = \frac{700 - 2.5}{5k\Omega} = 0.1395\,A$$

Applying KVL for loop ABCDA, $V_{AB} = 0.1395 \times 1 \times 10^3 + 2.5$

$= 142V$

$V_{TH} = V_{AB} = 142\,V$



Maximum power is transferred when $R_L = R_{TH}$.

Thus, $R_L = 800\Omega$

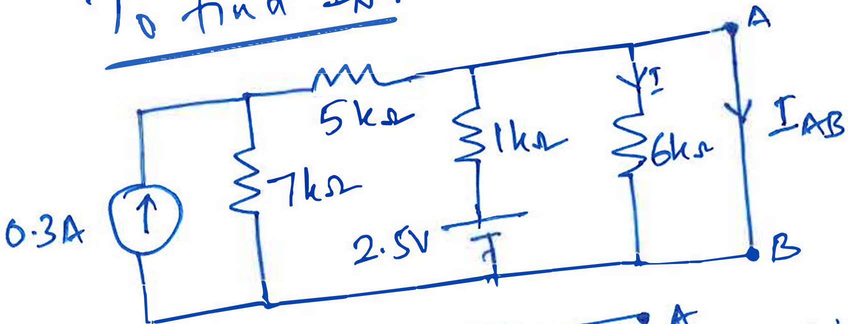
$$I_L = \frac{V_{TH}}{R_{TH}+R_L} = \frac{142}{800+800} = \underline{88.75\ mA}$$

$$V_L = I_L R_L = \underline{71V} \qquad P_L = V_L I_L = \underline{6.3\ W}$$

NORTON'S ~~EQUIVLA~~ EQUIVALENT

$R_N = R_{TH} = 800\Omega$

To find I_N:



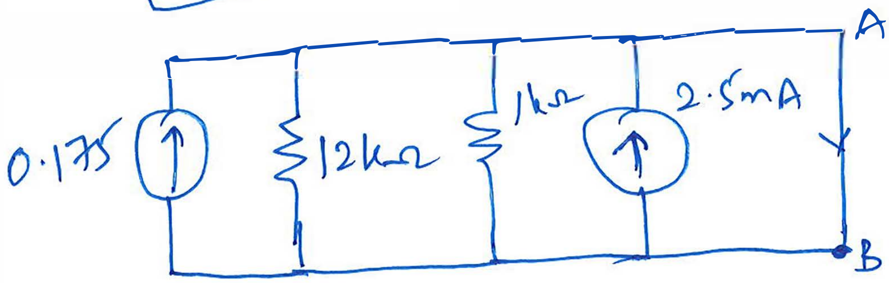
Using Source transformation

$$I_{S1} = \frac{2.5V}{1k\Omega} = 2.5mA$$

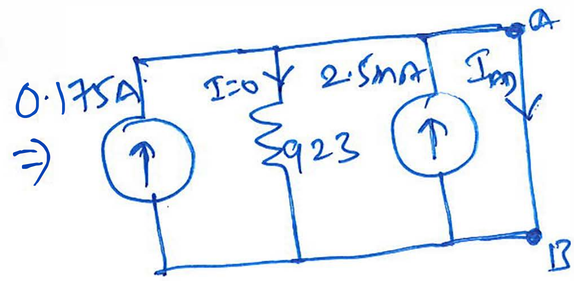
$$V_{S1} = 0.3 \times 7k = 2100V$$

Using Source transformation,

$$I_{S1} = \frac{2100}{12k} = 0.175$$

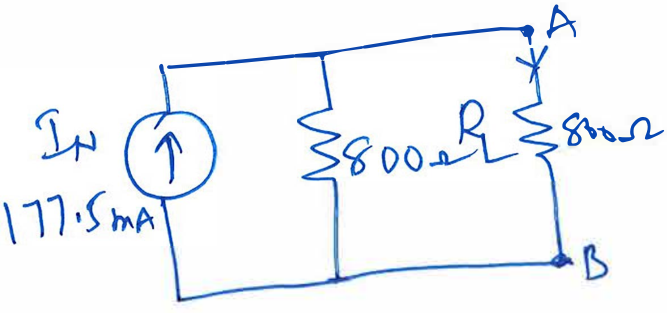


$\Rightarrow$



$$I_{AB} = 0.175A + 2.5mA = \underline{177.5\ mA}$$

$$I_N = I_{AB} = \underline{177.5\ mA}$$

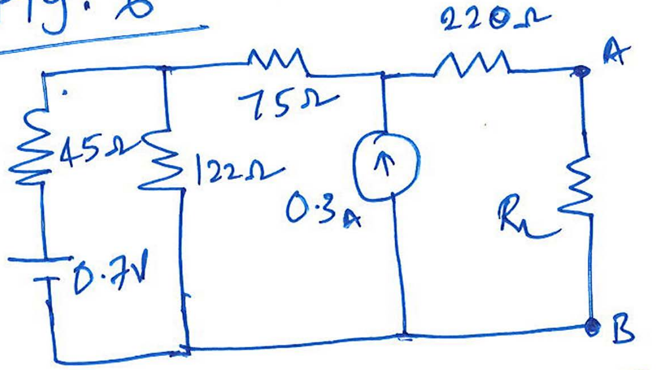


$$I_L = \frac{177.5m \times 800}{800+800} = 88.75\ mA$$

$$V_L = 71V$$

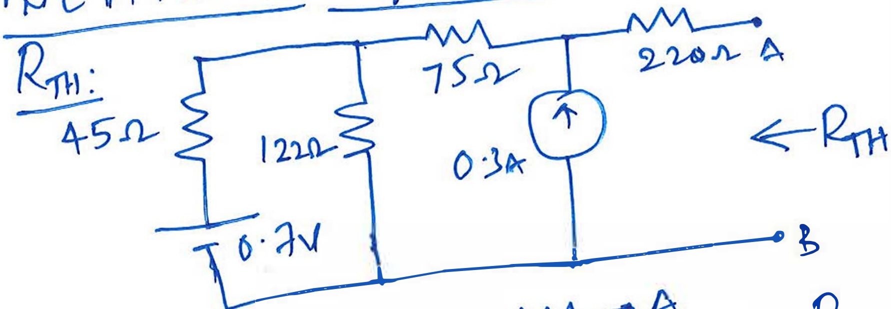
$$P_L = V_L I_L = \underline{6.3\ W}$$

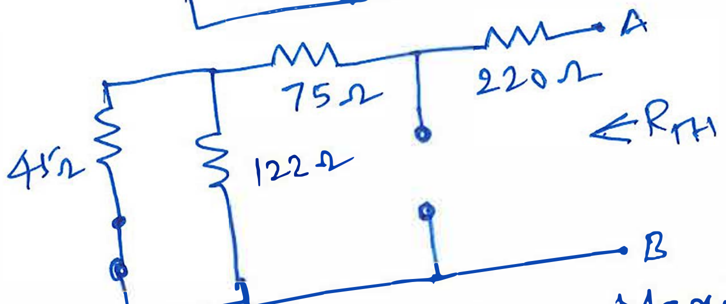
Fig. 6



THEVININ'S EQUIVALENT :

R_{TH}:



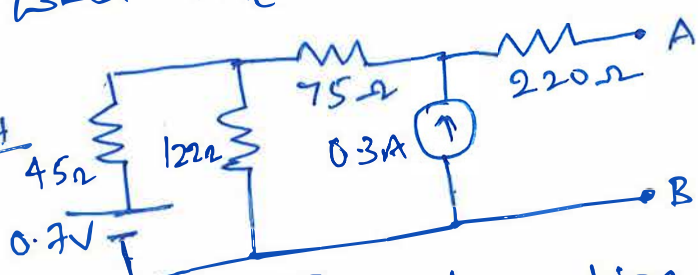


$R_{AB} = 220 + 75 + (45 \| 122)$

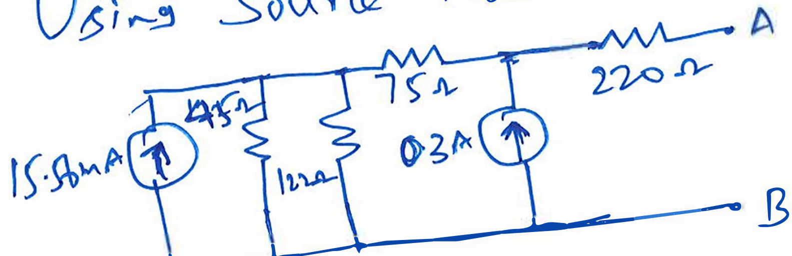
$= 323\ \Omega$

$\underline{R_{TH}} = R_{AB} = \underline{323\ \Omega}$. Maximum power is transferred when $R_L = R_{TH}$. $\underline{\underline{R_L = 323\ \Omega}}$

V_{TH}



Using Source Transformation $I_{s_1} = \dfrac{0.7V}{45\Omega} = 15.56mA$



$45 \| 122 = 33\Omega$

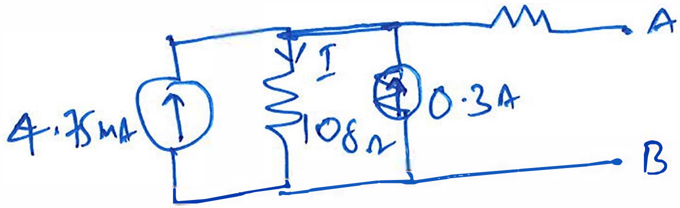
Using Source transformation,

$V_{s_1} = 15.56mA \times 33 = \underline{0.513V}$

Using Source transformation,

$$I_s = \frac{0.513}{33+75} = 4.75\,mA$$

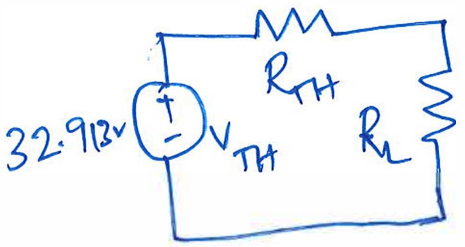
$$R_s = 108\,\Omega$$



$$I = 0.3 + 4.75\,mA$$
$$= 304.75\,mA$$

$$V_{AB} = I \times 108\,\Omega = \underline{32.913\,V}$$

$$V_{TH} = V_{AB} = 32.913\,V$$



$$R_{TH} = R_L = 323\,\Omega$$

$$I_L = \frac{V_{TH}}{R_{TH}+R_L} = 50.95\,mA$$

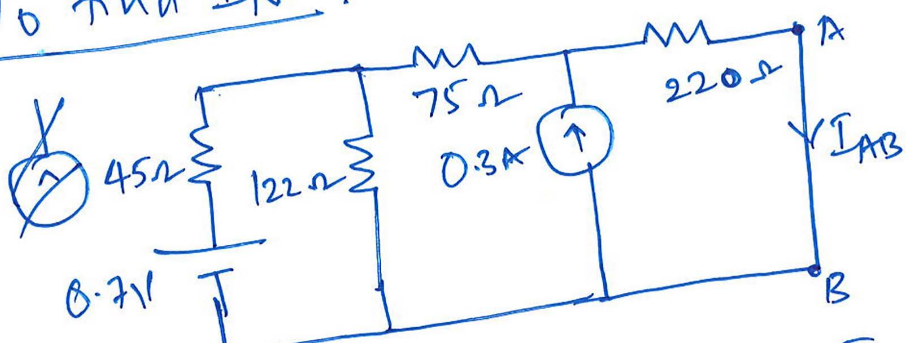
$$V_L = I_L \cdot R_L = 16.4565\,V$$

$$P_L = V_L \cdot I_L = \underline{0.8384\,W}$$

Norton's EQUIVALENT

$$R_N = R_{TH} = 323\,\Omega$$

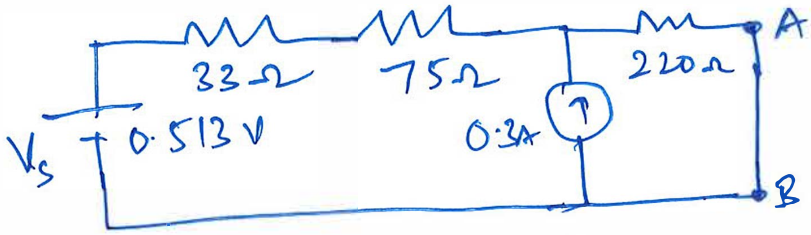
To find I_N:



Using source transformation, $I_{S1} = \frac{0.7V}{45\Omega} = 15.56\,mA$

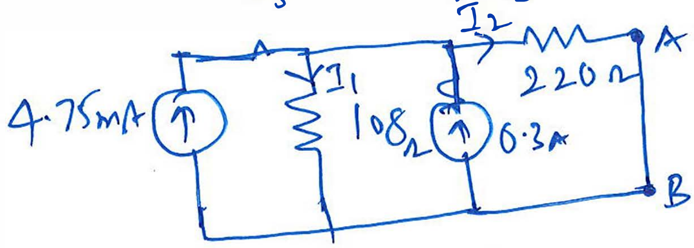
$45 \| 122 = 33\,\Omega$ $\quad R_{S1} = 33\,\Omega \quad I_{S1} = 15.56\,mA$

Using source transformation, $V_{S1} = I_{S1} R_1 = \underline{0.513V}$



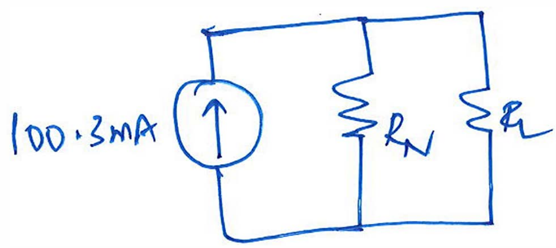
Using source transformation, $R_S = 33 + 75 = 108\Omega$

$I_s = V_s / R_s = 4.75\ mA$



$$I_2 = \frac{(0.3 + 4.75\times10^{-3})\times 108}{108+220} \approx 100.3\ mA$$

$I_N = I_{AB} = I_2 = 100.3\ mA$



$R_N = R_L = 330\Omega$

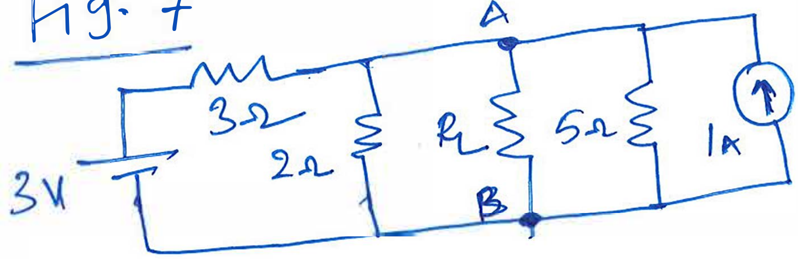
$I_L = 50.17\ mA$

$V_L = 16.56\ V$

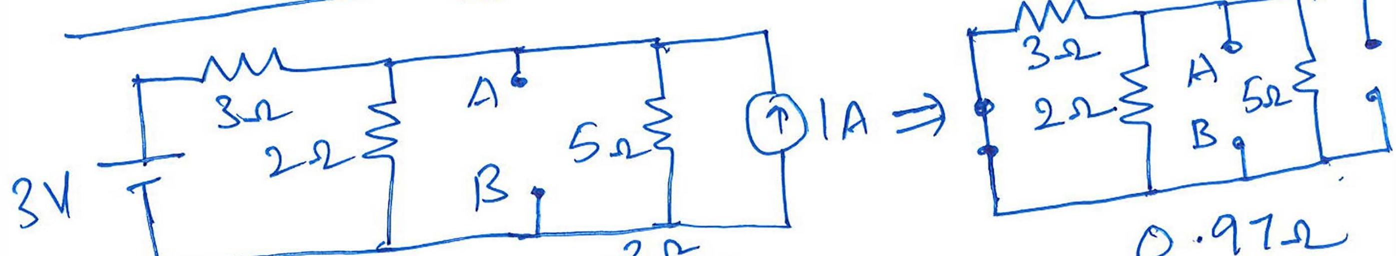
$P_L = 0.83\ W$

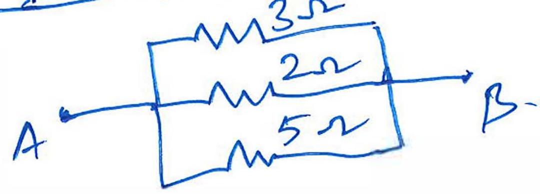
(small differences are due to rounding off errors)

Fig. 7



THEVININ'S EQUIVALENT:





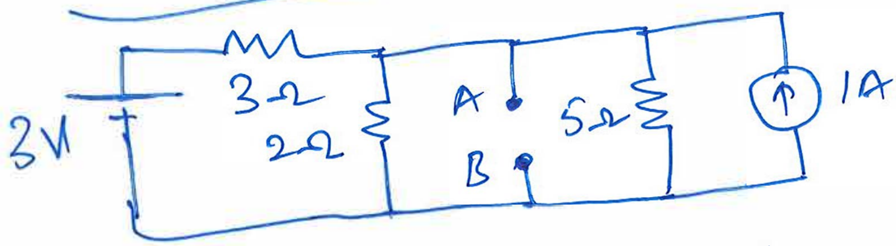
$R_{AB} = 0.97\Omega$

$R_{TH} = R_{AB} = 0.97\,\Omega$

Maximum power is transferred when

$R_L = R_{TH}$ Thus, $R_L = 0.97\,\Omega$

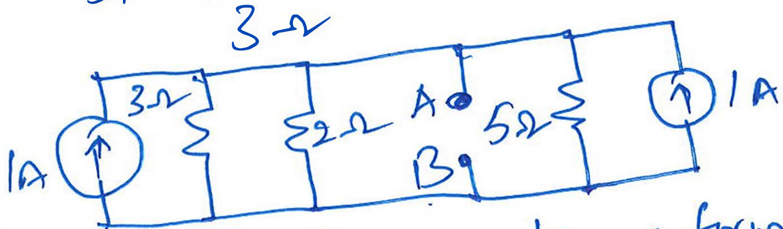
To find V_{TH}:



Using Source transformation,

$I_{S_1} = \dfrac{3V}{3\Omega} = 1A$

$3 \| 2 = 1.2\,\Omega$

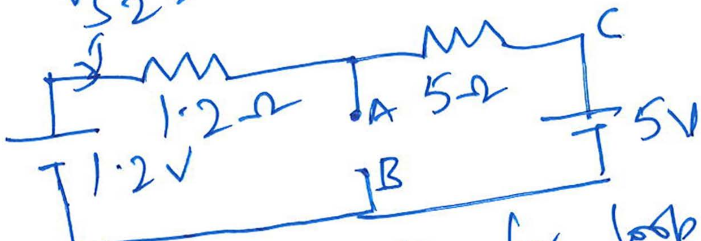


Using Source transformation, $V_{S_1} = 1 \times 1.2 = 1.2V$

$V_{S_2} = 1 \times 5 = 5V$

$I = \dfrac{1.2 - 5}{1.2 + 5}$

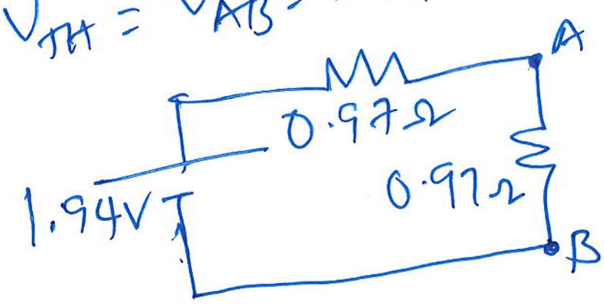
$= -0.613A$



Applying KVL for loop ACBA,

$V_{AB} = -5 \times 0.613 + 5 = 1.94V$

$V_{TH} = V_{AB} = 1.94V$



$I_L = \dfrac{1.94}{2 \times 0.97} = 1A$

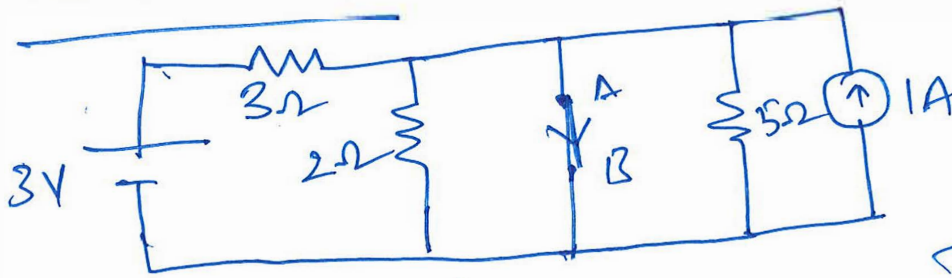
$V_L = 0.97V$

$P_L = V_L \cdot I_L = \underline{0.97W}$

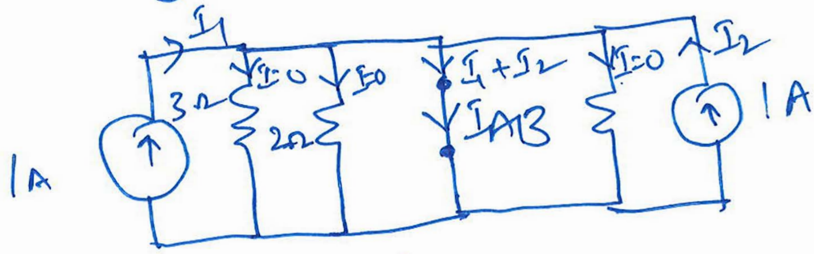
Norton's Equivalent

$R_N = R_{TH} = 0.97\,\Omega$

To find I_N:

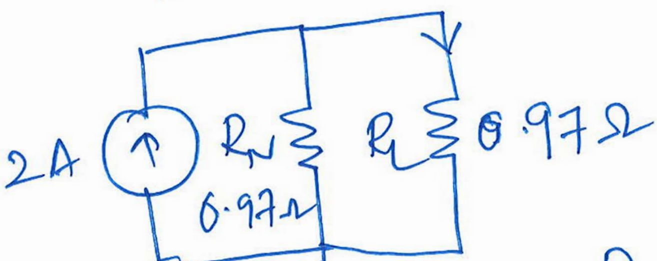


Using Source transformation. $I_{S_1} = \frac{3V}{3\Omega} = 1A$



$I_{AB} = I_1 + I_2 = 2A$

$I_N = I_{AB} = 2A.$



$I_L = \frac{2\times 0.97}{0.97+0.97} = 1A$

$V_L = I_L \cdot R_L = 0.97V$

$P_L = V_L I_L = \underline{0.97W}$